SPECTRAL ASYMPTOTICS OF SCHRÖDINGER OPERATORS WITH SINGULAR INTERACTIONS ON SURFACES

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In this talk I will survey some results of my doctoral thesis and various more recent developments in this field in which I was involved. The key object of consideration is the resolvent power difference

$$\mathsf{G} := (\mathsf{H} - \lambda)^{-m} - (-\Delta_{\rm free} - \lambda)^{-m},$$

where $-\Delta_{\rm free}$ is the self-adjoint free Laplacian in $L^2(\mathbb{R}^d)$ with $d \geq 2$, $\lambda \in \rho(\mathsf{H}) \cap \rho(-\Delta_{\rm free})$, $m \in \mathbb{N}$, and H is a self-adjoint Schrödinger operator in $L^2(\mathbb{R}^d)$ with one of the following three types of singular interactions.

- δ-interaction supported on a hypersurface $(\mathrm{codim} = 1)$.
- δ'-interaction supported on a hypersurface $(\mathrm{codim} = 1)$.
- δ-interaction supported on a surface with $\mathrm{codim} = 2$.

If the interaction support is compact and sufficiently smooth, and the interaction strength is bounded, then the operator G turns out to be compact. In many questions (such as *scattering theory*, *structure of the* ac-*spectrum*, *eigenvalue estimates*, *etc.*) it is of certain importance to refine the knowledge on the operator G. The results, which are going to be discussed, fall into one of the following four closely related classes.

- Asymptotic spectral estimates for G.
- Spectral asymptotics of G.
- Estimates of the remainder in the spectral asymptotics of G.
- Formulae for the trace of G whenever G is in the trace class.

In the proofs various tools and methods are used. Several of them deserve to be highlighted.

- Operator extension theory and Krein-type resolvent formulae.
- Pseudo-differential techniques.
- Spectral asymptotics of ψdo's on smooth manifolds.
- Properties of the trace.

These results are obtained in different combinations of co-authors jointly with my colleagues from TU Graz: J. Behrndt, C. Kühn, J. Rohleder, and also with G. Grubb (U. Copenhagen) and M. Langer (U. Strathclyde, Glasgow).