Symmetry decomposition of functions on compact semisimple Lie groups

J. Patera
Université de Montréal

The familiar decomposition
\[ f(x) = f_s(x) + f_a(x), \quad 0 \leq x \leq 1 \]
of a given function \( f(x) \) into its symmetric and antisymmetric parts,
\[ f_s(x) = \frac{1}{2}(f(x) + f(1-x)), \quad f_a(x) = \frac{1}{2}(f(x) - f(1-x)), \]
can be interpreted as the central decomposition of a class function \( f(x) \) on \( SU(2) \).

In the talk we describe central decomposition of class functions \( f(x_1, x_2, \ldots, x_n) \) on a compact semisimple Lie group \( G \) of rank \( n < \infty \) and of any type, into as many symmetry components as is the order of the center of \( G \). Such decomposition is either continuous if \( x_1, x_2, \ldots, x_n \in \mathbb{R} \), or discrete if the variables specify a point of an \( n \)-dimensional lattice \( L_M \) of symmetry compatible with \( G \) and of any density \( M \).

Examples of central decompositions of functions on \( SU(2) \times SU(2), SU(3), Sp(4), E(6) \), and some useful properties of the component functions will be pointed out.