A FABER-KRAHN INEQUALITY FOR THE ROBIN LAPLACIAN ON EXTERIOR DOMAINS

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We will discuss several generalizations of the Faber-Krahn inequality for the lowest eigenvalue of the Robin Laplacian with a negative boundary parameter on the exterior $\mathbb{R}^d \setminus \overline{\Omega}$ to a bounded, simply connected, smooth domain $\Omega \subset \mathbb{R}^d$. Some further generalizations for disconnected Ω's will also be discussed. Our main motivation is to go beyond more traditional bounded domains in eigenvalue optimization.

The ultimate goal is always to prove that the exterior of a ball maximizes the underlying lowest eigenvalue under a suitable constraint being imposed. In two dimensions, we constrain either the perimeter of Ω or its area. Constraining the area of $\partial\Omega$ leads in higher dimensions to an ill-posed optimization problem, as the large coupling asymptotics in PANKRASHKIN-POPOFF-16 shows. Instead, we constrain for $d \geq 3$ the ratio between a Willmore-type energy of $\partial\Omega$ and the area of $\partial\Omega$, assuming, additionally, that Ω is convex.

In the proofs we represent the lowest eigenvalue via the min-max principle on the level of quadratic forms expressed in suitably chosen coordinates on $\mathbb{R}^d \setminus \overline{\Omega}$. We make use either of standard parallel coordinates or of their modification worked out in PAYNE-WEINBERGER-61 and further refined in SAVO-01. The trickiest part of the proof is to find a proper test function.

These results are obtained in collaboration [KL-I, KL-II] with David Krejčiřík.

REFERENCES

[KL-I] D. Krejčiřík and V. Lotoreichik, Optimisation of the lowest Robin eigenvalue in the exterior of a compact set, *to appear in J. Convex Anal.*, `arXiv:1608.04896`.

[KL-II] D. Krejčiřik and V. Lotoreichik, Optimisation of the lowest Robin eigenvalue in the exterior of a compact set, II: non-convex domains and higher dimensions, *submitted*, `arXiv:1707.02269`.