Quasi-Hermitian Model with Point Interactions and Supersymmetry

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### Introduction

**Supersymmetry** in quantum mechanics with two Hermitian point interactions (at $x = 0$ and $x = l$) is allowed only for two special classes of models [1]. Are the systems with two $PT$-symmetric point interactions compatible with supersymmetry? What are the appropriate classes of boundary conditions? Are all energy levels real? Is it possible to construct positive metric operator $\psi$?

**$PT$-symmetric point interactions and SUSY**

- **General $PT$-symmetric point interactions** are characterized by boundary conditions [2]
  
  \[
  \left(\psi(0) - e^{i \theta} \psi(0)\right) = 0,
  \left(\psi(0) - e^{-i \theta} \psi(-0)\right) = 0.
  \]

- **Requirement of SUSY** restricts the ranges of parameters $b, c, \theta, \phi$

- **The only possible boundary conditions** are given by matrices $B_{\pm}$

\[
\Psi = \left(\begin{array}{c}
\psi(x) \\
\psi(-x)
\end{array}\right), \quad \Psi' = \left(\begin{array}{c}
\psi'(x) \\
-\psi'(x)
\end{array}\right), \quad B_{\pm} = \pm \left(\begin{array}{c}
\tan \phi & i \tan \phi
\end{array}\right).
\]

**Model of the type $(+ +)$**

- **Both interactions characterized by $B_{+}$ matrix**
- **Eigenvalues and eigenfunctions**
  
  \[
  E_n = \left(\frac{(n + 1)^2}{4}\right)^2, \quad n \in \mathbb{N}.
  \]

- **Supersymmetry is unbroken**
  
  \[
  \Theta = I - \frac{\beta_1}{\beta_1 + \beta_2} P^+ P + \frac{\beta_2}{\beta_1 + \beta_2} P^+ P, \quad P^+ \text{ are projectors}
  \]

  \[
  (P^+ \psi)(x) = \theta(\pm x) \psi(x), \quad (P^+ \psi)^* = (P^+ \psi)^*, \quad P^+ P^* = P^* P = 0
  \]

**Model of the type $(+ -)$**

- **Interactions characterized by $B_{-}$ at $x = 0$ and by $B_{+}$ at $x = l$**
- **Eigenvalues and eigenfunctions**
  
  \[
  E_n = \left(\frac{(n - 1)^2}{4}\right)^2, \quad n \in \mathbb{N}.
  \]

- **Supersymmetry is broken**
  
  \[
  \Theta = P^+ (O_1 + O_2) P^+ + P^+ (O_1 + O_2) P^+ - \frac{\beta_2}{\beta_1 + \beta_2} P^+ O_2 P^+, \quad O_{1,2} \text{ are projectors}
  \]

**References**