

The 14th international conference  
**Analytic and algebraic methods in physics**

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→ **THE BOOK OF ABSTRACTS** ←  
(the version of July 13, 2017)

(alphabetical ordering)

**Anna Allilueva**

**Waves on geometric graphs**

We describe certain properties of wave equations on geometric graphs. In particular, we study equations corresponding to different types of Laplacians and corresponding reflections on the vertices. We discuss the distribution of wave energy; in particular we show that in certain cases this distribution is governed by the location of eigenspaces of the corresponding unitary operator. For strongly localized solutions, this operator reduced to reflection from a subspace.

# Petr Ambrož

## Voronoi Tilings of Quasicrystals

Quasicrystals are materials whose diffraction pattern reveals rotational symmetries forbidden in periodic patterns. Experimentally observed were symmetries of order 8, 10 and 12. One of possible models of quasicrystals are point sets constructed using the well-known cut-and-project method. Local configurations of such point sets can be characterized via corresponding Voronoi tiling. As quasicrystals with decagonal symmetry were already treated, we consider quasicrystals with 12-fold rotational symmetry and we describe their distinct Voronoi tiles based on the size of the acceptance window of said cut-and-project method. For one particular size of the dodecagonal acceptance window we obtain a Voronoi tiling composed of only two prototiles.

**Fabio Bagarello**

**kq-representation for non-Hermitian position and momentum operators, and bi-coherent states**

We show how the Zak kq-representation, originally used in many-body theory, can be adapted to deal with pseudo-bosons, and under which conditions this can be done. By means of this representation we prove completeness of a discrete set of bi-coherent states constructed out of pseudo-bosonic operators. The case of Riesz bi-coherent states is analyzed in detail.

# Holger Cartarius

## Open many-body quantum systems with topologically non-trivial phases

Non-Hermitian systems with PT symmetry can possess purely real eigenvalue spectra if the eigenstates are PT symmetric as well. At the edges of multi-site many-body systems topologically protected states can appear, which lead to complex relations with PT-symmetric potentials. We study one-dimensional bosonic and fermionic systems, in which topologically nontrivial edge states appear, and investigate their behaviour in relation to balanced gain and loss of the probability amplitude modelled by complex PT-symmetric potentials. Typically the edge states are not PT symmetric since their probability amplitude is localised only on one side of the system, however, a particle-hole symmetry can ensure their PT symmetry. For a many-particle system a dynamical approach with a master equation provides a more direct modelling of the in- and outcoupling of particles. Thus, we investigate our models with a master equation in Lindblad form. It is shown that the dynamics of the density matrix follows the predictions of stationary calculations using PT-symmetric potentials. In particular it is found that there is a clear distinction in the dynamics between the topologically different cases known from the stationary eigenstates.

Work done together with Marcel Klett, Felix Dangel, Marcel Wagner, Dennis Dast and Guenter Wunner

**Francisco M. Fernández and Javier Garcia**

**Highly accurate calculation of the real and complex eigenvalues of one-dimensional anharmonic oscillators**

We draw attention on the fact that the Riccati-Padé method developed some time ago enables the accurate calculation of bound-state eigenvalues as well as of resonances embedded either in the continuum or in the discrete spectrum. We apply the approach to several one-dimensional models that exhibit different kind of spectra. In particular we test a WKB formula for the imaginary part of the resonance in the discrete spectrum of a three-well potential.

# Daniel Hook

## Behavior of eigenvalues in a region of broken PT symmetry

PT-symmetric quantum mechanics began with a study of the Hamiltonian  $H = p^2 + x^2(ix)^\epsilon$ . When  $\epsilon \geq 0$ , the eigenvalues of this non-Hermitian Hamiltonian are discrete, real, and positive. This portion of parameter space is known as the region of unbroken PT symmetry. In the region of broken PT symmetry,  $\epsilon < 0$ , only a finite number of eigenvalues are real and the remaining eigenvalues appear as complex-conjugate pairs. The region of unbroken PT symmetry has been studied but the region of broken PT symmetry has thus far been unexplored. This paper presents a detailed numerical and analytical examination of the behavior of the eigenvalues for  $-4 < \epsilon < 0$ . In particular, it reports the discovery of an infinite-order exceptional point at  $\epsilon = -1$ , a transition from a discrete spectrum to a partially continuous spectrum at  $\epsilon = -2$ , a transition at the Coulomb value  $\epsilon = -3$ , and the behavior of the eigenvalues as  $\epsilon$  approaches the conformal limit  $\epsilon = -4$ .

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Results of collaboration with

Carl M. Bender, Nima Hassanpour, S. P. Klevansky, Christoph Sanderhauf and Zichao Wen, published, on May 15th, 2017, in Phys. Rev. A 95, 052113.

# A.M. Ishkhanyan

## Bi-confluent Heun solutions of the Schrödinger equation

We present the five six-parametric Lamieux-Bose potentials for which the general solution of the one-dimensional Schrödinger equation is written in terms of the bi-confluent Heun functions [1,2]. To derive the confluent hypergeometric reductions of this family of potentials, we construct an expansion of the solutions of the bi-confluent Heun equation in terms of the Hermite functions. The series is governed by a three-term recurrence relation between successive coefficients of the expansion. We examine the restrictions that are imposed on the involved parameters in order that the series terminates thus resulting in closed-form finite-sum solutions of the bi-confluent Heun equation. We further identify a particular conditionally integrable potential for which the involved bi-confluent Heun function admits a four-term finite-sum expansion in terms of the Hermite functions [3]. This is an infinite well defined on a half-axis. We present the explicit solution of the one-dimensional Schrödinger equation for this potential and discuss the bound states supported by the potential. We derive the exact equation for the energy spectrum and construct a highly accurate approximation for the bound-state energy levels.

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Results of collaboration with T. A. Ishkhanyan

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[1] A. Lamieux and A.K. Bose, "Construction de potentiels pour lesquels l'équation de Schrödinger est soluble", Ann. Inst. Henri Poincaré A 10, 259-270 (1969).

[2] A.M. Ishkhanyan and V.P. Krainov, "Discretization of Natanzon potentials".

[3] T.A. Ishkhanyan and A.M. Ishkhanyan, "Solutions of the bi-confluent Heun equation in terms of the Hermite functions", Ann. of Phys. (2017),

DOI: <https://doi.org/10.1016/j.aop.2017.04.015>, arXiv:1608.02245 [quant-ph].



# Vladimir V. Konotop

## CPT-symmetric coupled nonlinear Schrödinger equations

There will be considered two models of the coupled nonlinear Schrödinger equation which are symmetric with respect to simultaneous time-reversal (T), spatial-inversion (P), and field exchange (C). The first model describes the spin-orbit coupled Bose-Einstein condensate with loading of atoms in one hyperfine state and removing from the other one. The second model describes two optical waveguides, one with gain and another with loss, whose coupling is highly dispersive. For both models there will be discussed features of the (CPT)-symmetry breaking, the existence of nonlinear modes (the first model) and solitons (the second model), as well as their stability. Both works are done in collaboration with Y. V. Kartashov and D. A. Zezyulin.

[1] Kartashov, Y. V., Konotop, V. V., & Zezyulin, D. A. (2014). CPT -symmetric spin-orbit - coupled condensate. *EPL (Europhysics Letters)*, 107, 50002.

[2] Zezyulin, D. A., Kartashov, Y. V., & Konotop, V. V. (2017). CPT-symmetric coupler with intermodal dispersion, *42(7)*, 1273-1276.

**Ali Mostafazadeh**

**Nonlinear Generalization of Spectral Singularity and Transfer Matrix**

We introduce nonlinear extensions of the notions of spectral singularity and transfer matrix for nonlinearities with a compact support. We discuss how some basic relations of laser physics can be obtained from the condition of the presence of a linear or nonlinear spectral singularity. In particular we obtain the laser threshold condition and output intensity for the TE and TM modes of a slab laser and predict that the lasing is forbidden for TM modes with an incident angle exceeding the Brewster's angle. We also discuss applications of our nonlinear transfer matrix in the investigation of spectral singularities and unidirectional invisibility in the presence of confined nonlinearities.

**Satoshi Ohya**

**Exactly Solvable Bound-State Problems and Lie Algebras  $so(2,1)$ ,  
 $so(3)$ , and  $iso(2)$**

I shall revisit very old bound-state problems in quantum mechanics. I shall focus on five specific examples: Coulomb, trigonometric Rosen-Morse, Manning-Rosen, hyperbolic Rosen-Morse, and attractive Liouville potentials, all of whose bound state problems are known to be exactly solvable. I show that in these problems the energy eigenfunctions as well as energy eigenvalues are all obtained by means of the unitary representations of the Lie algebras  $so(2,1)$ ,  $so(3)$ , or  $iso(2)$ .

# Fabio Rinaldi

## Gaussian potential is essentially solvable

In this presentation we expand the preliminary findings showing that the one-dimensional Schrödinger equation with an attractive Gaussian potential is essentially solvable in the sense that its eigenvalues, whose number depends on the magnitude of the coupling parameter but is always greater than or equal to one, can be determined by diagonalising a trace class operator, whose integral kernel can be suitably expressed in p-space, and solving the corresponding number of transcendental equations to get the solutions as functions of the coupling constant.

**Etsuo Segawa**

**Discrete-time quantum walks induced by quantum graphs**

I introduce discrete-time quantum walks induced by quantum graphs with some boundary conditions. In this talk, we discuss on a resonance of the quantum walks.

# **Andrei Shafarevich**

## **Laplacians and wave equations on polyhedral surfaces**

We study properties of Laplacians and wave equations on polyhedral surfaces. These equations appear, particularly in the problem of long waves' scattering on point obstacles. We describe different types of Laplacians, their kernels (spaces of harmonic functions), trace formulas and wave fronts for localized solutions of wave equations.

# Teoman Turgut

## Uniqueness of the ground state for Lee model

We define an oversimplified version of the Lee model, in which a two level system is interacting with bosonic particles on a three dimensional manifold. We show that this system can be understood by an associated operator, so called the principal operator, after removing an infinity (mass renormalization). We indicate that this operator contains all the information about the dynamics and the spectrum is bounded from below. We explain a method to show that there is a well defined self-adjoint Hamiltonian defined by this operator. We sketch the proof that on a compact manifold the lowest eigen-state of this operator is unique, using the theory of positivity improving semigroups.